NBV-003-016402

NBV-003-016402 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT-4002

(Integration Theory)

Faculty Code : 003

Subject Code : 016402

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Answer all questions.

(2) Each question carries **14** marks.

1 Answer any **seven** questions. Each question carries two marks : **2×7=14**

(i) Define σ-finite measure on a measurable space (X, A) and give an example of a σ-finite measure.

(ii) If μ is the Lebesgue measure on $[0,1]$ and v is the atomic measure on [0, 1] concentrated at $\frac{1}{2}$ then $\left(0, \frac{1}{2}\right)$ is a ________ set w.r.t. $\mu - \gamma$. Justify.

(iii) Define complete measure on a measurable space and give an example of a complete measure.

(iv) If γ is a signed measure on $(X, \mathscr{A})$ then $A \in \mathscr{A}$ is a null set w.r.t. γ iff $|\gamma|(E) =$ ________. Justify.

(v) True or False ? Justify. If γ is a signed measure on $(X, \mathscr{A}), A \in \mathscr{A}$ and $\gamma(A) = 0$ then A is null set w.r.t. γ.

(vi) Define μ^*-measurable subset of a set x with outer measure μ^*. Prove that every $E \subset X$ with $\mu^* E = 0$ is μ^*-measurable.

(vii) Prove that the product of two σ-finite complete measures is σ-finite.

(viii) If $f \in L^1(X, \mathscr{A}, \mu)$ then prove that $f(x)$ is finite are $x \in X$.

(ix) If x is a to topological space and $E \subset X$ then find supp (x_E).

(x) Give an example of a space which is locally compact but not compact. Justify.

2 Answer any **two** questions : **2×7=14**

(a) State and prove Hahn decomposition theorem.

(b) Define Jordan decomposition of a signed measure on a measurable space and prove that it is unique.

(c) If μ_1, μ_2 are two measures on a measurable space $(X, \mathscr{A})$ and at least one of them is finite then prove that $\mu_1 - \mu_2$ is a signed measure on $(X, \mathscr{A})$.

3 (a) Define the concept of a measure absolutely continuous w.r.t. another measure. If γ is the lebesgue measure on $\mathbb{R}$ and γ is the counting measure on $\mathbb{R}$ then prove that $\gamma << \mu$. If $\mu << \gamma$? Justify. **7**

(b) Define mutually singular measures on a measurable space $(X, \mathscr{A})$ and give an example of mutually singular measures. **7**

OR

3 (c) If μ is the counting measure on a countable set X then prove that $L^p(\mu) \cong l^p, \forall\ 1 \le p \le \infty$. **7**

(d) State, without proof, caratheodary extension theorem. Prove that the assumption "μ is σ-finite" in the theorem can not be dropped. **7**

4 Answer any **two** questions : **2×7=14**

(a) Define Baire measure on the real line. Prove that the cumulative distribution function "F" of a finite signed measure on the real line is bdd, monotonically increasing and $\lim\limits_{x \to -\infty} f(x) = 0$.

(b) If μ is a measure on an algebra $\mathcal{A}$ of subsets of a set X and μ^* is the outer measure on X induced by μ then prove that every $E \in \mathcal{A}$ is μ^*-measurable.

(c) Give an example of a compact Hausdorff space X

$st\, Ba(X) \underset{\neq}{\subset} Bo(X).$

5 Answer any **two** questions : **2×7=14**

(a) If X is a locally compact separable metric space then prove that $Bo(X) = Ba(X)$.

(b) State, without proofs, Fubini's theorem and Tonelli's theorem.

(c) Define σ-compact set in a locally compact Hausdorff space. Prove that every σ-compact open set in a locally compact Hausdorff space is a Baire set.

(d) Give an example of baire measure on a locally compact Hausdorff space which is not regular. Justify.